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A RELATION BETWEEN THE APPROXIMATED VERSIONS OF MINIMUM SET COVERING, MINIMUM VERTEX COVERING AND MAXIMUM INDEPENDENT SET

CAHIER N° 121
octobre 1993

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received : March 1993.

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Résumé

Soit ρ une constante universelle représentant le rapport d'approximation d'un algorithme approché pour les instances du problème du stable maximum vérifiant $\frac{9}{20}n \leq \alpha(G) \leq \frac{11}{20}n$, où G est un graphe d'ordre n et de nombre de stabilité $\alpha(G)$. Supposons qu'il existe un algorithme approché de rapport constant pour le problème de la couverture minimum d'ensembles. Il existe alors un algorithme polynomial approché pour le problème de la couverture minimum de sommets avec un rapport majoré par $2 - \frac{9}{10}\rho + \varepsilon$ avec ε arbitrairement petit.

mots-clés: Problème NP-complet, algorithme polynomial approché, couverture d'ensembles, couverture de sommets, stable

A relation between the approximated versions of minimum set covering, minimum vertex covering and maximum independent set

Abstract

Let ρ be a universal constant denoting the approximation ratio of a polynomial time approximation algorithm for the instances of the independent set problem with $\frac{9}{20}n \leq \alpha(G) \leq \frac{11}{20}n$, where G is a graph of order n and stability number $\alpha(G)$. Let finally suppose the existence of a (universally) constant-ratio-polynomial-time-approximation-algorithm for set covering problem. Then there exists a polynomial time approximation algorithm for vertex covering problem with a ratio bounded above by $2 - \frac{9}{10}\rho + \varepsilon$ for an ε arbitrarily small.

keywords: NP-complete problem, polynomial time approximation algorithm, set covering, vertex covering, independent set

1 Introduction

Given a graph $G = (V, E)$ of order n, a vertex cover is a subset $V' \subseteq V$ such that for each edge $uv \in E$ at least one of u and v belongs to V' and the minimum vertex covering problem (VC) is to find a vertex cover of minimum size.
Also, given a collection $\mathcal{S}$ of subsets of a finite set C, a set cover for C is a subcollection $\mathcal{S}'$ of $\mathcal{S}$ such that every element of C belongs to at least one member of $\mathcal{S}'$ and the minimum set covering problem (SC) is to find a set cover of minimum size.
Finally, given a graph $G = (V, E)$, an independent set is a subset $S \subseteq V$ such that not any two nodes in V' are linked by an edge in G and the maximum independent set problem (IS) is to find an independent set of maximum size.

One of the most interesting theoretical problems in the complexity theory is to be able to "transfer" approximation results (positive, negative, or conditional) from an NP-complete problem to another via reductions preserving approximations ratios or to condition the existence (or the improvement) of existing approximation performances for some problems on the existence (or the improvement) of approximation performances for other ones.
VC is a famous combinatorial problem for which we know a polynomial time approximation algorithm (PTAA) with a ratio equal to 2, namely the maximal matching algorithm [5,8]. But, up to now all researchers have failed either to find another approximation algorithm with better performance guarantees. On the other hand, recently, some researchers ([1]) have proved that VC does not admit a polynomial time approximation schema unless P = NP. In the light of this remarkable result, the evaluation of a value constituting the lower bound for the approximation ratio of VC, or an improvement of the known approximation ratio for VC would be of great theoretical interest. Concerning the improvement of this ratio, we mention here the works of R. Bar-Yehuda and S. Even ([2,3]). Their results concern an improvement of VC's approximation ratio from 2 to $2-\epsilon$, but for an $\epsilon = \frac{\log \log n}{2 \log n}$ ([3]) wich tends to ∞ with n.
In this paper, we propose a conditional method for the improvement of VC's ratio for an absolute constant ϵ, by considering VC as a restriction of SC. In fact we link, from an approximability point of view, three optimization problems, the VC, SC and IS. We prove then, that a sufficient condition for the improvement of VC's approximation ratio is the simultaneous existence of an approximation algorithm for SC and an approximation algorithm for IS on graphs for which holds $\frac{9}{20}n \leq \alpha(G) \leq \frac{11}{20}n$, where $\alpha(G)$ denotes the stability number of the graph G and the two approximation algorithms are supposed of constant approximation guarantees.

In fact, every instance I of SC characterized by two sets $\mathcal{S}$ and C ($\mathcal{S} = \{s_1, \ldots, s_n\}$ denoting the family of the subsets of the set $C = \{c_1, c_2, \ldots, c_m\}$, where n, m are the cardinalities of $\mathcal{S}$ and C respectively), can be represented by a bipartite graph $B = (S, C, E)$, called the characteristic graph B of I, where the vertex set S denotes the family $\mathcal{S}$, the vertex set C the elements of the set C and $E = \{s_i c_j : c_j \in s_i\}$.
Every instance I of VC is expressed in terms of a graph $G = (V, E)$, which can be equivalently represented by a bipartite graph $B_G = (V, E, E')$ where E', the edge set of B_G, contains the pairs $v_i e_j$ such that e_j is incident to v_i in G.
Clearly the instances of VC are exactly the instances of SC where every element of C

is contained in exactly two subsets of $\mathcal{S}$, or equivalently, in the characteristic graph of these instances of SC the degrees of the C-vertices are equal to 2. Hence we can treat every instance of VC as an instance of SC, by considering G or equivalently B_G as the characteristic graph of this instance. Consequently, the result of [1] is valid for SC also.

By supposing next that a PTAA for SC exists, we derive a PTAA that provides a ratio for VC of value strictly smaller than 2. Our method consists, given an instance of SC, in constructing a new larger instance of the problem in which the cardinality of a set covering is a power of the cardinality of the solution in the initial instance.
This construction is performed by means of a kind of operation on bipartite graphs, called composition, where, given two bipartite graphs $B_i = (S_i, C_i, E_i)$ and $B_j = (S_j, C_j, E_j)$, someone can construct the bipartite graph $B_i * B_j = B_{ij} = (S_{ij}, C_{ij}, E_{ij})$ with $S_{ij} = S_i \times S_j$, $C_{ij} = C_i \times C_j$, and $E_{ij} = \{s_{mn}c_{kl} : s_m c_k \in E_i \wedge s_n c_l \in E_j\}$, where the operator $\times$ denotes the cartesian product.
We denote by $B^i = (S_i, C_i, E_i)$ the graph obtained by the following inductive schema:

$$\begin{aligned} B^1 &= B \\ B^i &= B * B^{i-1}. \end{aligned} \tag{1}$$

In what follows we will suppose that ρ', ρ are universal constants representing approximation ratios for SC and a family $\mathcal{G}$ of instances of IS respectively. This family is defined as

$$\mathcal{G} = \{G : \frac{9}{20}n \le \alpha(G) \le \frac{11}{20}n\}$$

where n is the order of the graph G and $\alpha(G)$ its stability number (cardinality of a maximum independent set of G). We can suppose that the IS algorithm $\mathcal{A}''$, solving approximately the instances of the family $\mathcal{G}$, when applied to graphs not contained in $\mathcal{G}$ provides either solutions with ratio smaller than ρ ($\rho \le 1$) or non feasible solutions. Also we will denote by τ the cardinality of a minimum vertex cover.

2 The result

Theorem. *Let ρ be the approximation ratio of a polynomial time approximation algorithm $\mathcal{A}''$ solving independent set on graphs in $\mathcal{G}$ and let us suppose the existence of a polynomial time approximation algorithm $\mathcal{A}$ of (universally) constant approximation ratio for set covering. Then there exists a polynomial time approximation algorithm for vertex covering with a ratio bounded above by $2 - \frac{9}{10}\rho + \varepsilon$, for a ε arbitrarily small.*

Whenever $\alpha(G) \le \frac{9}{20}n$, we have obviously $\tau \ge \frac{11}{20}n$ and consequently, since any minimal vertex covering is at most of cardinality n (recall that by n we denote the order of G), any suboptimal algorithm for VC has an approximation ratio bounded above by

$$\frac{n}{\frac{11}{20}n} = \frac{20}{11} < 1.82.$$

Thus, the main part of the proof concerns the case $\alpha(G) \ge \frac{9}{20}n$.
In what follows we assume the existence of a PTAA $\mathcal{A}$ with approximation ratio ρ' (absolute constant) for SC which provides us with a solution T'_i of cardinality τ'_i for B^i

(inductive schema of equation (1)), by means of which we shall derive a solution T' of cardinality τ' for B_G (or equiv. for G).

In fact, given the graph B^i we can see the set C_i as the union of $m = |C|$ groups of cardinality $m^{i-1} = |C_{i-1}|$, every group C_{i-1} as the union of $m = |C|$ groups of cardinality $m^{i-2} = |C_{i-2}|, \ldots$, every C_2 group as the union of $m = |C|$ groups of cardinality $m = |C|$ (the same correspondence holds also for the set S_i). Moreover for every j, set C_{j-1} of the graph B^j is "seen" by two S_{j-1} groups of B^j. This is clear by the definition of B^j, since every c-vertex of B_G is "seen" by two s-vertices of B_G.

Consequently, a solution T' for VC on a graph $G = (V, E)$ can be obtained by the algorithmic schema described by algorithm 1.

For example for the case $i = 2$, let us denote by M_i ($| M_i |= \mu_i$), $i = 1, 2, \ldots, n$ the subsets of T'_2 in the S-group S_i.

Then the solution T' is obtained by taking one of the subsets of T'_2 of minimum cardinality which covers a C-group of B^2 (the minimum being taken over the distinct C-groups), or more formally:

$$T' = \{M_i \cup M_j : |M_i \cup M_j| = \min_{k,l=1,2,\ldots,n} |\{M_k \cup M_l : s_k s_l \in E(G)\}|\}.$$

In fact, since in B every vertex c_k "sees" two s-vertices, s_i, s_j, then by the construction of the graph B^i (equation (1)), vertex c_k corresponds to a C-group C_k of B^i receiving edges issued only from two S-groups S_i, S_j of B^i corresponding to the vertices s_i, s_j of B_G, and consequently the part of the solution T'_i covering the elements of the group C_k is contained in the groups S_i, S_j.

We are now well prepared to continue the proof.

Let us suppose that a graph G instance of VC is given. We apply $\mathcal{A}'$ (algorithm 1) to G. Step [1] of $\mathcal{A}'$ serves to treat the cases where $\alpha(G) \leq \frac{9}{20}n$. Let us now estimate the size τ'_k of the solution T'_k.

We examine the following two cases corresponding to steps [5] and [6] of algorithm $\mathcal{A}'$ respectively:

(a) $\forall i \leq k, \tau'_i \geq n\frac{\tau'_{i-1}}{2}$.

(b) $\exists i \leq k, \tau'_i \leq n\frac{\tau'_{i-1}}{2}$.

(a) $\forall i \leq k, \tau'_i \geq n\frac{\tau'_{i-1}}{2}$.

Here, we have to examine two subcases concerning $\alpha(G)$.

(a1) $\alpha(G) \geq \frac{11}{20}n$.

(a2) $\frac{9}{20}n \leq \alpha(G) \leq \frac{11}{20}n$.

(a1) Let τ, m be the cardinalities of a minimum vertex cover and a maximum matching in G respectively.

[1] Given the graph $G = (V, E)$, obtain a maximal matching on G. Store as candidate solution for VC the vertices incident to the edges of the maximal matching just obtained.

[2] Construct the characteristic graph $B = (S, C, E')$, with $S = V, C = E$.

[3] Fix two arbitrarily small universal constants ϵ, ε and construct the graph B^k (inductive schema of equation (1), where k is the smallest integer for which

$$\rho'^{\frac{1}{k}} \leq 1 + \epsilon \,. \tag{2}$$

where

$$\epsilon \leq \frac{\varepsilon}{2 - \frac{9}{10}\rho} \tag{3}$$

[4] Execute $\mathcal{A}$ on the instance of SC represented by B^k, let $T'_k(|T'_k| = \tau'_k)$ be the obtained solution.

[5] If $\forall i \leq k, \tau'_i \geq n\frac{\tau'_{i-1}}{2}$ then construct a solution $T'(|T'| = \tau')$ for G (or equivalently for B) by taking the subset of T'_k of minimum cardinality which covers a C-group embedded in a C_2 group, embedded ..., embedded in C_k (the minimum being taken over the distinct C-groups).
Also, execute $\mathcal{A}''$ to G and let α' be the cardinality of the obtained solution S'. If S' is feasible and moreover $\alpha' \geq \frac{9}{20}\rho n$ then store the set $T'' = V \setminus S'$ as candidate solution. Go to step [7].

[6] If $\exists i \leq k, \tau'_i \leq n\frac{\tau'_{i-1}}{2}$, then let P (Q) be the set of the S_{i-1}-groups of B^i with more (less) than $\frac{\tau'_{i-1}}{2}$ subsets of T'_i.
Construct the graph $BG = (P, Q, E')$, which is the bipartite graph resulting from G by removing all the edges between the members of $P(G)$ and obtain a maximum matching M on BG. Let PS, PE (QS, QE) the saturated and the exposed vertices in P (Q) with respect to M.
Start from set QE and take into account the members of PS adjacent to the members of QE (let call this set by PS'). Take into account the mates of the set PS' (let call this set by QS'). Then augment PS' by inserting in this set all the vertices adjacent to the members of QS' that are not already in PS'. Augment also QS' by taking into account the mates of the vertices recently added to PS' and repeat this procedure until no more vertices can be added to PS'.
Let G' and G'' the subgraphs of G induced by the sets $PS' \cup QS' \cup QE$ and $(P \setminus PS') \cup (Q \setminus (QS' \cup QE))$ respectively. Take as solution of G' the set PS'.
Go to step [1] and replace G by G''.

[7] The final solution for G is the smallest between
(i) the set obtained in step [1],
(ii) the union of T' (obtained in step [5]) with the union of the sets PS' created from the (eventually multiple) executions of step [6] and
(iii) the union of T'' (obtained in step [5]) with the union of the sets PS' created from the (eventually multiple) executions of step [6].

Algorithm 1. Algorithm $\mathcal{A}'$ associating solutions for SC to solutions of VC.

Given that ([4]) $\alpha(G) + \tau = n$ and $m \leq \tau$, we have

$$m \leq \tau \leq \frac{9}{20}n. \tag{4}$$

We know also ([4,5]) that given a maximum matching M, the set of vertices incident to the edges of M constitute a solution T' for VC of cardinality $\tau' \leq 2m$, thus by using equation (4)

$$\tau' \leq \frac{9}{10}n$$

and by taking into account the fact that the exposed vertices[1] of a graph with respect to a maximal matching form an independent set of the graph we obtain immediately such a set of cardinality

$$\alpha' \geq \frac{n}{10}.$$

We have thus

$$\begin{aligned} n &= \tau' + \alpha' \\ &\geq \tau' + \frac{n}{10} \\ &\geq \tau' + \sum_{j \in I\!N^+} \frac{\tau'}{10^j} \quad \text{or} \\ n &\geq \frac{10}{9}\tau'. \end{aligned} \tag{5}$$

By an easy induction on i and using the hypothesis of case **(a)** and equation (5), we conclude that

$$\tau'_k \geq \left(\frac{5}{9}\right)^{k-1} \tau'^k. \tag{6}$$

(a2) In this case, if α' is the cardinality of the independent set obtained from $\mathcal{A}''$, it verifies

$$\begin{aligned} \frac{\alpha'}{\alpha} &\geq \rho \quad \text{or} \\ \alpha' &\geq \rho\alpha \\ &\geq \frac{9}{20}\rho n. \end{aligned}$$

By using arguments similar to those for case **(a1)**, we obtain

$$\begin{aligned} n &\geq \frac{20}{20 - 9\rho}\tau' \quad \text{or} \\ \tau'_k &\geq \left(\frac{10}{20 - 9\rho}\right)^{k-1} \tau'^k. \end{aligned} \tag{7}$$

This concludes case **(a)**.

[1] We use here the terminology of [8] where exposed vertices (with respect to a maximal matching M) are called the vertices that are not saturated by the edges of M; also given an edge uv of M, u (v) is called the mate of v (u).

(b) $\exists i \leq k, \tau'_i \leq n\frac{\tau'_{i-1}}{2}$.

Of course, the inequality $\tau'_i \leq n\frac{\tau'_{i-1}}{2}$ imposes in B^i the existence of some S_{i-1}-groups with less than $\frac{\tau'_{i-1}}{2}$ subsets if the solution T'_i (set Q).
As we have already seen, in B^i there are $n = |S|$ S_{i-1}-groups, each one of these n groups representing a vertex of G when seen with respect to the whole graph B^i. Thus we have equivalently a partition of $S(G)$ in two sets P and Q, the set Q being an independent set of G. The argument is that as the S-groups that form Q contain each one less than $\frac{\tau'_{i-1}}{2}$ members of T'_i, the existence of a C_i-group of B^i (equiv. an edge of G) "seen" in common by two groups of Q would lead to a smaller solution τ'_{i-1}.
Moreover, in what concerns sets PE, QE, if QE is empty, then the constraint $\tau'_i \leq n\frac{\tau'_{i-1}}{2}$ is not true. This is because if the whole of Q is saturated then all the groups in $PS \cup Q$ of the graph B^i contain at means more than $\frac{\tau'_{i-1}}{2}$ elements. Moreover the members of PE contain T'_i-groups of cardinality greater than $\frac{\tau'_{i-1}}{2}$. On the other hand if PE is empty ($P = PS$) or M is perfect, then the optimal solution for G is found. The arguments: as $PS = P$ is saturated by the matching M, the mates of this set is set QS and P is a solution for G (complement of an independent set, i.e $S \setminus Q$) and moreover in every graph the cardinality of a vertex covering is greater or equal to the cardinality of a maximum matching ([4]), the minimum over all the possible solutions is found.
Thus the sets PS, PE, QS, QE provided by the execution of step [6] of $\mathcal{A}'$ are all non-empty.

Of course the fact that M is a maximum matching implies that there will never be a vertex member of PE added in PS' during the described procedure. In fact during step [6], we proceed by creating sets of alternating paths. If for instance we suppose that by this construction we attain a member of PE this means exactly that we have discovered an augmented path and of course the hypothesis that M is a maximum matching is contradicted. Also the fact that there are no more vertices that can be added in PS' during step [6] of algorithm 1 implies that all the members of the so formed QS' are adjacent exclusively to the members of PS' formed throughout the procedure. At the end of step [6] of $\mathcal{A}'$ we have a partition of the vertices of G into two sets namely PS' and $P' = P \setminus PS'$.
We claim that PS' is an optimal solution of VC in G'. Clearly, PS' is a solution for G', as its members are adjacent to all other vertices ($QS' \cup QE$) of G'. Moreover this solution is optimal for G'. The arguments: the way we have constructed PS' implies that all the members of this set are endpoints of edges contained in M. Moreover all the other edges emanate from those vertices. Finally the edges added for completing the graph G are edges between the members of PS'. Thus the cardinality of PS' is exactly the cardinality of a matching in G' and thus the solution induced by PS' is minimum ([4]).
Also by the way we have conceived step [6] of $\mathcal{A}'$, there are no edges between the members of $QS' \cup QE$ and the vertices of the graph G'' for which all the vertices of the set $Q \setminus (QS' \cup QE)$ are saturated.

Finally algorithm $\mathcal{A}'$ produces a partition of G say $G'_1, G'_2, \ldots, G'_\ell$ such that G'_i, $i < \ell$

are polynomially solved and G_ℓ is either polynomially solved or admits the constraint $\tau_i' \geq n^{\frac{\tau_{i-1}'}{2}}$ where now τ_i', n and τ_{i-1}' concern G_ℓ, for which case **(a)** is applicable. Let us denote by G' the union $\bigcup_{i \leq \ell-1} G_i$ of the graphs produced by the (eventually multiple) execution of step [6] and G'' the graph G_ℓ.

For this partition of G into the graphs G', G'' we can prove that *the approximation ratio ρ of an algorithm solving approximately the VC in G is smaller than the approximation ratio ρ_2 of an algorithm solving approximately the VC in G''.*

Really, let us consider the independent set Q_1 associated to the solution T_1' ($|T_1'| = \tau_1'$) of G'. We denote by T_1 ($|T_1| = \tau_1$) the optimal solution for G'. We have already proved that $T_1' = T_1$ ($\tau_1' = \tau_1$).

Let T_2' ($|T_2'| = \tau_2'$) and T_2 ($|T_2| = \tau_2$) be the approximate and optimal solution respectively for G'' and let $\frac{\tau_2'}{\tau_2} \leq \rho_2$ for a fixed constant ρ_2.

As there are no edges between members of Q_1 and vertices of G'', there are no more edges between Q_1 and the independent set Q_2 associated to T_2', thus if T' is a solution of G then $T' = T_1' \cup T_2'$.

Obviously, if T ($|T| = \tau$) is the optimal solution of G, then $\tau \geq \tau_1 + \tau_2$ because T has, eventually, to cover not only the edges of G_1 and G_2 but also the edges between G_1 and G_2, thus we have

$$\begin{aligned} \frac{\tau_1'}{\tau_1} &= 1 \quad \text{and} \\ \frac{\tau_2'}{\tau_2} &\leq \rho_2 \quad \text{or} \\ \rho &= \frac{\tau'}{\tau} \leq \frac{\tau_1' + \tau_2'}{\tau_1 + \tau_2} = \frac{\tau_1 + \tau_2'}{\tau_1 + \tau_2} \leq \rho_2. \end{aligned} \tag{8}$$

The last line of step [6] of algorithm 1 implies the application of steps [1] $\div$ [6] of the algorithm on G''.

It remains now to explore the approximation ratio for VC induced by solutions for SC found after the *kth* composition of G'' (step [5] of algorithm 1). In any case (see equations (6) and (7)), the cardinalities of the solutions obtained in this step are of the form

$$\tau_k' \geq \beta^{k-1} \tau'^k \tag{9}$$

where $\beta > \frac{1}{2}$ and equal either to $\frac{5}{9}$ (equation (6)) or to $\frac{10}{20-9\rho}$ (equation (7)).

Moreover, for the optimal solutions τ, τ_k of G, B^k respectively, we have

$$\tau^k \geq \tau_k. \tag{10}$$

From equations (9), (10), and the fact that the approximation algorithm for SC $\mathcal{A}$ has approximation ratio ρ', we have

$$\begin{aligned} \rho' &\geq \frac{\tau_k'}{\tau_k} \geq \beta^{k-1} \left(\frac{\tau'}{\tau}\right)^k \quad \text{or} \\ \frac{\tau'}{\tau} &\leq \frac{1}{\beta^{\frac{k-1}{k}}} \rho'^{\frac{1}{k}}. \end{aligned}$$

We have already seen that if the composition of algorithm $\mathcal{A}'$ is performed on G'', the solution for G obtained in step [7] approaches the optimal one within an error smaller than the one for the solution of G'' (equation (8)).
As $\frac{5}{9} \geq \frac{10}{20-9\rho}$ we have (see also equation (2))

$$\begin{aligned} \frac{\tau'}{\tau} &\leq (1+\epsilon)\frac{20-9\rho}{10} \quad \text{and by equation (3)} \\ \frac{\tau'}{\tau} &\leq 2 - \frac{9}{10}\rho + \varepsilon. \end{aligned}$$

Thus in any case the solutions obtained from the execution of step [7] of $\mathcal{A}'$ for VC are always less than $2 - \frac{9}{10}\rho + \varepsilon$ and as we can choose ε to be arbitrarily small, the approximation ratio for VC tends to $2 - \frac{9}{10}\rho < 2$.

3 Discussion

The result of section 2 has brought to the fore an aspect of the complex relation, concerning their approximation behaviour, between three known and difficult combinatorial optimization problems. We think that such results in a theoretical level contribute to produce a deeper knowledge of the approximation mechanisms in the class NP-complete. On the other hand they could help us in deeper understanding of the properties of this class as well as of the relations between its problems, relations that are not exhausted in the fact that the existence of an exact polynomial algorithm for one of them would imply the existence of such an algorithm for all of the problems. Moreover, the investigation of this type of relation, from a "practical" point of view could produce immediate positive or negative results for some of the problems concerned. If for example, the conditions of the theorem concerning IS and SC were true a new improved algorithm for VC would be immediately found.
Unfortunately, this "practical" significance of the above result is not valid. In fact, in [7] (see also [6]) Lund and Yannakakis have proved strong negative result for SC's approximability: SC cannot be approximated with ratio $c \log m$ for any $c < \frac{1}{4}$ unless NP $\subseteq$ DTIME$[n^{\text{poly} \log n}]$ (conjecture weaker than P = NP but highly improbable). On the other hand, the approximability of IS in the class $\mathcal{G}$, even if such a result has not be proven yet, is very improbable[2]. For one more time, in theoretical computer science it is very frequent, we have produce theoretical results, we have eventually increased the number of open questions, without, unfortunately, increasing the number of the answers.

Acknowledgement. I want to thank Lalo Fernandez de la Vega for his helpful comments and our fruitful discussions on this work. I thank also Rajeev Motwani for his very carefull reading of an early version of the paper and for our discussions-by-e_mail.

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