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A GENERALIZATION OF THE INDISCERNIBILITY RELATION FOR ROUGH SETS ANALYSIS OF QUANTITATIVE INFORMATION ¹

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GÉNÉRALISATION DE LA RELATION D'INDISCERNABILITÉ POUR L'ANALYSE À L'AIDE DES ENSEMBLES APPROXIMATIFS D'INFORMATION QUANTITATIVE

Résumé

La théorie des ensembles approximatifs est un outil formel pour l'analyse de connaissance acquise par l'expérience. La connaissance est représentée par un ensemble de données sous la forme d'une table appelée système d'information. Les lignes de la table correspondent aux objets et les colonnes aux attributs. L'idée de l'ensemble approximatif consiste à approximer un ensemble d'objets par une paire d'ensembles appelés approximation inférieure et supérieure. La définition de ces approximations découle d'une relation d'indiscernabilité entre objets. Les objets sont décrits par des attributs qui sont de nature soit qualitative, soit quantitative. Dans le cas d'attributs quantitatifs, la relation d'indiscernabilité est définie après partition de l'échelle réelle en un nombre fini d'intervalles. Les bornes de ces intervalles, étant plus ou moins arbitraires, peuvent influencer le résultat de l'analyse de connaissance à l'aide des ensembles approximatifs. Pour appréhender cette influence, nous considérons des intervalles qui se chevauchent en leur extrémités et introduisons les relations d'indiscernabilité faible et forte. Nous généralisons ensuite les approximations inférieures et supérieures, les mesures de la qualité d'approximation ainsi que le concept de la règle de décision.

Mots-clés : Théorie des ensembles approximatifs, analyse de connaissance, système d'information, relation d'indiscernabilité entre objets.

A GENERALIZATION OF THE INDISCERNIBILITY RELATION FOR ROUGH SETS ANALYSIS OF QUANTITATIVE INFORMATION

Abstract

Rough sets theory is a formal tool for analysis of knowledge gained by experience. The knowledge is represented by a data set organized in a table called information system. Rows of the table correspond to objects and columns to attributes. The idea of the rough set consists in approximation of a set of objects by a pair of sets called lower and upper approximation. The definition of the approximations follows from an indiscernibility relation between objects. Objects are described by attributes of qualitative or quantitative nature. In the case of quantitative attributes, the indiscernibility relation has been defined after partition of the real scale into a finite number of intervals. The bounds of the intervals are more or less arbitrary and may influence the result of the rough sets analysis. In order to capture this influence, we consider overlapped intervals and introduce for them a strict and a weak indiscernibility relation. Then, we generalize the lower and upper approximations, the measures of the quality of approximation and the concept of decision rules.

Keywords : Rough sets theory, knowledge analysis, information system, indiscernibility relation among objects.

1. INTRODUCTION

Rough sets theory provides a formal tool for transforming a data set, being a record of an experience, into knowledge understood as a family of classification patterns related to a specific part of the real world. One of factors hindering revelation of the classification patterns from data representing an experience is indiscernibility caused by granularity of the representation.

Rough sets theory, which deals with the granularity problem, has been proposed in early 80's by Pawlak [6,7] and quickly gained a wide acceptance because of successful applications (cf.[2,4,5,9,11,13]). The main advantage of rough sets theory is that it does not need any preliminary or additional information about data (like probability in statistics, basic probability number in Dempster-Shafer theory, grade of membership, or the value of possibility in fuzzy sets theory). Another advantage is that inconsistencies in the data base are not corrected; instead, produced classification rules are categorized into certain and possible.

Rough sets have been compared to fuzzy sets, sometimes with an idea of making them competitive models of an imperfect knowledge. Such a comparison is, however, ill-posed because indiscernibility and vagueness are two different faces of an imperfect knowledge (cf.[1,8]). Basically, rough sets embody the idea of indiscernibility between objects in a set, while fuzzy sets model the ill-definition of the boundary of a sub-class of this set. In other words, rough sets are calculus of partitions, while fuzzy sets are continuous generalization

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of set-characteristic functions. Taking an example from image processing, rough sets theory is concerned by the size of pixels used to approximate contours, while fuzzy sets theory is concerned by the level of gray between black and white. Comparison of the rough sets theory with the related approaches have been made in [1,3,8,11,14].

The observation that one cannot distinguish objects (clients, states, cases, etc.) on the basis of given information about them is the starting point of the rough sets philosophy. In other words, imperfect information causes indiscernibility of objects. The indiscernibility relation induces an approximation space made of equivalence classes of indiscernible objects. A rough set is a pair of a lower and an upper approximation of a set in terms of these equivalence classes. The concept of an information system is used to construct the approximation space.

The information system is the 4-tuple $S = \langle U, Q, V, \rho \rangle$, where U is a finite set of objects, Q is a finite set of attributes, $V = \bigtimes_{q \in Q} V_q$ and V_q is a domain of the attribute q , and $\rho: U \times Q \rightarrow V$ is a total function such that $\rho(x, q) \in V_q$ for every $q \in Q$, $x \in U$, called information function. Any pair (q, v) , $q \in Q$, $v \in V_q$ is called descriptor in S .

The information system is in fact a finite data table, columns of which are labelled by attributes, rows are labelled by objects and the entry in column q and row x has the value $\rho(x, q)$. Each row in the table represents the information about an object in S .

For example, a data file concerning clients applying for a

credit in a bank is an information system. Let us suppose that the clients have submitted their applications in the form of dossiers including information about their previous financial results, present state and business plans. So, they can be described by a set of attributes defined on the basis of dossiers. When the time of pay back comes, some clients are able to repay and others are not, thus the ability of repayment makes partition of the set of clients into two classes. An analysis of the experience gained by the bank should answer then the following questions:

- what is the quality of approximation of particular classes of clients by the set of attributes; if it is low, it means that some extra attributes had to be taken into account when making decision about granting the credit,
- what is the minimal subset of attributes which ensures the same quality of approximation as the whole set; if there are several minimal subsets, those attributes which enter the intersection of minimal subsets represent a non-removable part of the information while other attributes from minimal subsets are exchangeable,
- what are classification (decision) rules in terms of attributes from a chosen minimal subset; the rules are "certain" if the attributes univocally assign the clients to particular classes, and they are "possible" otherwise; the rules explain the relationship between description of clients and their ability of paying back the debt, so they may be useful to support decisions about granting new credits.

One may get answers on the above questions using the rough sets analysis. Many analogical cases have been described in the book devoted to applications of the rough sets theory [11].

The attributes describing objects may have a qualitative or a quantitative nature. In the latter case, the original domain of an attribute is a subset of the real line, e.g. value of a fixed capital or value of sales are values from a real interval. In practice, however, the original domain of a quantitative attribute is partitioned into few subintervals, qualified as subintervals of low, medium, high, etc. values. This partition renders objects easily comparable. Two objects are considered as indiscernible with regard to such an attribute if their values $\rho(x,q)$ belong to the same subinterval. The bounds of the subintervals are called norms. The norms are more or less arbitrary, they follow from some conventions. As shown in [12], definition of the norms may influence the result of the rough sets analysis. In order to capture this influence, we propose in this paper to consider overlapped subintervals and to distinguish between a strict and a weak indiscernibility relation.

Traditionnally [6], two objects $x,y \in U$ are indiscernible with regard to quantitative attribute q , if their values $\rho(x,q)$ and $\rho(y,q)$ are identical or belong to the same subinterval. We propose to enlarge the subintervals of attribute q from each side by a given threshold so that the subintervals overlap (cf. Fig. 1). Then, the two objects will be considered as strictly indiscernible with regard to q , if their values $\rho(x,q)$ and $\rho(y,q)$ belong to only one enlarged subinterval which is the

same for both values. They will be considered as weakly indiscernible if the values $\rho(x,q)$ and $\rho(y,q)$ belong to at least one enlarged subinterval and at least one is the same for both values. Indeed, for any $x \in U$, $\rho(x,q)$ may belong to one or two subintervals - in the latter case it takes value in an overlapping zone and in the former case, outside this zone. For example, objects x_2 and x_3 in Fig. 1 are strictly indiscernible while objects x_1, x_2, x_3 and x_4 are weakly indiscernible. Of course, strictly indiscernible objects are also weakly indiscernible, but not conversely. The difference between the quality of approximations defined for strict and weak indiscernibility relations gives an insight into an effect of overlapping. Let us mention that another generalization of the rough sets approach trying to deal with missing and/or imprecise data in the information systems has been made in [14].

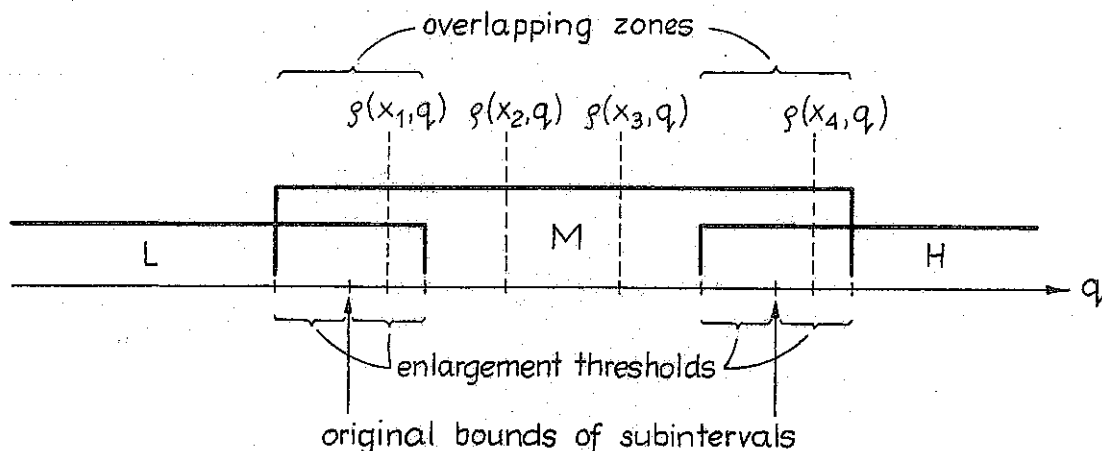


Fig. 1. Overlapping subintervals L, M, H of quantitative attribute q

This paper is an extended version of [10]. In the next section, we formally introduce the strict and weak indiscernibility relations. In two following sections, we generalize the lower and upper approximations and the measures of the quality of approximation. Then, we generalize the dependency of attributes and the notion of decision rules. An illustrative example and conclusions end the paper.

2. STRICT AND WEAK INDISCERNIBILITY RELATIONS

Let $S = \langle U, Q, V, \rho \rangle$ be an information system and $q \in Q$ a quantitative attribute. The domain V_q is a real interval $[b_q^L, b_q^U]$. Let split this interval into m_q subintervals $[b_{qi}^L, b_{qi}^U]$, $i=1, \dots, m_q$. Of course, $b_{qi}^L = b_{q(i-1)}^U = b_{qi}$, $i=1, \dots, m_q$. Let us also define a threshold $t_{qi}(b_{qi}) \geq 0$ for each bound b_{qi} . We will translate the original values $\rho(x, q)$ to, so called, coded values $\rho^c(x, q)$ being sets defined as

$$\rho^c(x, q) = \{v_i^c : \rho(x, q) \in [b_{qi}^L - t_{qi}(b_{qi}), b_{qi}^U + t_{qi}(b_{qi})], i \in \{1, \dots, m_q\}\}$$

where v_i^c is a code of subinterval i . Let us notice that $\rho^c(x, q)$ may have one or two elements only.

The qualitative attributes take also coded values $\rho^c(x, q)$ but they are singletons. System S with coded values of attributes is called coded information system and denoted by S^c .

Let $P \subseteq Q$ and $x, y \in U$. We say that objects x and y are strictly indiscernible in S^c by set of attributes P iff

$$\rho^c(x, q) = \rho^c(y, q) \quad \forall q \in P$$

$$\text{and } \text{card} [\rho^c(x, q)] = \text{card} [\rho^c(y, q)] = 1 \quad \forall q \in P.$$

Objects x and y are weakly indiscernible in S^c by set of attributes P iff

$$\rho^c(x, q) \cap \rho^c(y, q) \neq \emptyset \quad \forall q \in P.$$

Thus, every $P \subseteq Q$ generates two binary relations on U , called strict and weak indiscernibility relations, denoted by $IND^s(P)$ and $IND^v(P)$, respectively. Let us notice that $IND^s(P) \subseteq IND^v(P)$, and $IND^s(P)$ and $IND^v(P)$ are ordinary equivalence relations iff $IND^s(P) = IND^v(P)$ (partition of U). Indiscernibility classes of relation $IND^s(P)$ (resp. $IND^v(P)$) are called P^s -elementary (resp. P^v -elementary) sets in S^c . The union of all P^v -elementary sets is equal to U while the union of all P^s -elementary sets may be smaller than U .

The family of all indiscernibility classes of relation $IND^s(P)$ (resp. $IND^v(P)$) in U is denoted by $U|IND^s(P)$ (resp. $U|IND^v(P)$).

$Des_P(X^s)$ denotes the description of indifference class (P^s -elementary set) $X^s \in U|IND^s(P)$ in terms of the coded values of the attributes from P , i.e.:

$$Des_P(X^s) = \{(q, v_i^c) : v_i^c = \rho^c(x, q), i \in \{1, \dots, m_q\}, \\ \text{for every } x \in X^s \text{ and } q \in P\}$$

Similarly, for $X^v \in U|IND^v(P)$,

$$Des_P(X^v) = \{(q, v_i^c) : v_i^c \in \rho^c(x, q), i \in \{1, \dots, m_q\}, \\ \text{for every } x \in X^v \text{ and } q \in P\}$$

3. GENERALIZED APPROXIMATION OF SETS

In order to evaluate how well the sets $\{Des_P(X^s) : X^s \in U|IND^s(P)\}$ and $\{Des_P(X^v) : X^v \in U|IND^v(P)\}$ describe objects of a set $Y \in U$, we shall use the following concepts:

$\underline{P}^s Y = \bigcup X^s : \{X^s \in U \mid \text{IND}^s(P) \wedge X^s \subseteq Y\}$ - P^s -lower approximation of Y
in S^c (P^s -positive region of Y)

Set $\underline{P}^s Y$ is the set of all strictly indiscernible objects of U which can certainly be classified as belonging to Y , using attributes from P .

$\bar{P}^s Y = \bigcup X^s : \{X^s \in U \mid \text{IND}^s(P) \wedge X^s \cap Y \neq \emptyset\}$ - P^s -upper approximation of Y in S^c

Set $\bar{P}^s Y$ is the set of all strictly indiscernible objects of U which can possibly be classified as belonging to Y , using attributes from P .

$\text{Bn}_P^s(Y) = \bar{P}^s Y - \underline{P}^s Y$ - P^s -boundary of Y in S^c
(P -doubtful region of Y)

It is not possible to determine whether an object in $\text{Bn}_P^s(Y)$ belongs to Y solely on the basis of descriptions of the P^s -elementary sets.

Similarly,

$\underline{P}^v Y = \bigcup X^v : \{X^v \in U \mid \text{IND}^v(P) \wedge X^v \subseteq Y\}$ - P^v -lower approximation of Y
in S^c (P^v -positive region of Y)

$\bar{P}^v Y = \bigcup X^v : \{X^v \in U \mid \text{IND}^v(P) \wedge X^v \cap Y \neq \emptyset\}$ - P^v -upper approximation of Y in S^c

$\text{Bn}_P^v(Y) = \bar{P}^v Y - \underline{P}^v Y$ - P^v -boundary of Y in S^c
(P -doubtful region of Y)

Thus, $\underline{P}^v Y$ (resp. $\bar{P}^v Y$) is the set of all weakly indiscernible objects of U which can certainly (resp. possibly) be classified as belonging to Y , using attributes from P .

Let us notice, that there exist the following relations

between strict and weak approximations and the approximations based on the ordinary indiscernibility:

$$\underline{P}^s Y \subseteq \underline{P} Y \subseteq \underline{P}^v Y$$

$$\bar{P}^s Y \subseteq \bar{P} Y \subseteq \bar{P}^v Y$$

Let $\mathcal{X} = \{X_1, X_2, \dots, X_n\}$ be a classification of U , i.e. $X_i \cap X_j = \emptyset$ for every $i, j \leq n, i \neq j$, $\bigcup_{j=1}^n X_j = U$. X_j are called classes of \mathcal{X} .

By P^s -lower (resp. P^s -upper) approximation of \mathcal{X} in S^c we mean sets $\underline{P}^s \mathcal{X} = \{\underline{P}^s X_1, \underline{P}^s X_2, \dots, \underline{P}^s X_n\}$ (resp. $\bar{P}^s \mathcal{X} = \{\bar{P}^s X_1, \bar{P}^s X_2, \dots, \bar{P}^s X_n\}$). Similar definition exists for P^v -lower and P^v -upper approximations of \mathcal{X} in S^c .

4. MEASURES OF THE QUALITY OF APPROXIMATION HANDLING THE EFFECT OF OVERLAPPING

Let \mathcal{X} be a classification of U (for example given by an expert) in S^c . In order to evaluate how well the set of attributes $P \subseteq Q$ strictly or weakly approximates the classification \mathcal{X} , we shall use the following measures:

- the accuracy of strict or weak approximation of \mathcal{X} by P in S^c , respectively

$$\pi_P^s(\mathcal{X}) = \sum_{j=1}^n \text{card} [\underline{P}^s X_j] / \sum_{j=1}^n \text{card} [\bar{P}^s X_j]$$

$$\pi_P^v(\mathcal{X}) = \sum_{j=1}^n \text{card} [\underline{P}^v X_j] / \sum_{j=1}^n \text{card} [\bar{P}^v X_j]$$

- the quality of strict or weak approximation of \mathcal{X} by P in S^c , respectively

$$\gamma_P^s(\mathcal{X}) = \sum_{j=1}^n \text{card} [\underline{P}^s X_j] / \text{card} [U]$$

$$\gamma_P^v(\mathcal{X}) = \sum_{j=1}^n \text{card} [\underline{P}^v X_j] / \text{card} [U]$$

Let us notice that all these measures take values from interval $[0,1]$.

Since $\bar{P}^s X_j \subseteq \bar{P}^v X_j$, the difference $\bar{P}^v X_j - \bar{P}^s X_j$ is the boundary of X_j in S^c composed of P^v -elementary sets with ambiguous descriptions. The ambiguity follows from an overlapping of subintervals to the extent of thresholds t_{qi} , $i=1, \dots, m_q$, $q \in P$. This boundary is called P^t -boundary of X_j in S^c . Of course, if $t_{qi} = 0$ for all i and $q \in P$, then the P^t -boundary is empty.

Thus, the following measure can be used as an indicator of an influence of overlapping on the quality of approximation of classification \mathcal{X} :

$$\zeta_P(\mathcal{X}) = \text{card} [\bigcup_{j=1}^n (\bar{P}^v X_j - \bar{P}^s X_j)] / \text{card} [U]$$

$\zeta_P \in [0,1]$, and the smaller it is the lesser is the influence of overlapping.

The indicator says what is the ratio of the number of objects misclassified by P because of the overlapping and the number of all objects in S . For this reason, it can be useful in sensitivity analysis of rough classification with quantitative attributes (cf.[12]).

5. GENERALIZED DEPENDENCY OF ATTRIBUTES

Discovering dependencies between attributes is of primary importance in the rough sets approach to data analysis.

We will say that set of attributes $R \subseteq Q$ depends strictly on

the set of attributes $P \subseteq Q$ in S^c (denotation $P \xrightarrow{s} R$) iff $IND^s(P) \subseteq IND^s(R)$. Similarly, $R \subseteq Q$ depends weakly on $P \subseteq Q$ in S^c (denotation $P \xrightarrow{w} R$) iff $IND^w(P) \subseteq IND^w(R)$.

Another important issue is that of attribute reduction, in such a way that the reduced set of attributes provides the same quality of strict or weak approximation of classification \mathcal{X} as the original set of attributes. The minimal subset of attributes $R \subseteq P \subseteq Q$ such that $\gamma_P^s(\mathcal{X}) = \gamma_R^s(\mathcal{X})$ will be called \mathcal{X}^s -reduct of P (or, simply, s-reduct if there is no ambiguity in the understanding of \mathcal{X}). Similarly, we can define \mathcal{X}^w -reduct of P (or w-reduct).

Let us notice that a coded information system may have more than one \mathcal{X}^s -reduct and/or \mathcal{X}^w -reduct. $RED_{\mathcal{X}}^s(P)$ (resp. $RED_{\mathcal{X}}^w(P)$) is the family of all \mathcal{X}^s -reducts (resp. \mathcal{X}^w -reducts) of P . Intersection of all \mathcal{X}^s -reducts of P (resp. \mathcal{X}^w -reducts) will be called the \mathcal{X}^s -core (resp. \mathcal{X}^w -core) of P , i.e. $CORE_{\mathcal{X}}^s(P) = \bigcap RED_{\mathcal{X}}^s(P)$ and $CORE_{\mathcal{X}}^w(P) = \bigcap RED_{\mathcal{X}}^w(P)$. The \mathcal{X}^s -core (resp. \mathcal{X}^w -core) of Q is the set of the most characteristic attributes which cannot be eliminated from S^c without decreasing the quality of strict (resp. weak) approximation of classification \mathcal{X} .

6. GENERALIZED DECISION RULES

An information system can be seen as decision table assuming that $Q = C \cup D$ and $C \cap D = \emptyset$, where C are called condition attributes, and D , decision attributes.

Coded decision table $DT^c = \langle U, C \cup D, V, \rho^c \rangle$ will be called strictly deterministic iff $C \xrightarrow{s} D$ and, otherwise, strictly non-deterministic. Similarly, S^c will be called weakly

deterministic iff $C \xrightarrow{v} D$ and, otherwise, weakly non-deterministic. The strictly or weakly deterministic decision table uniquely describes the decisions to be made when some conditions are satisfied. In the case of a (strictly or weakly) non-deterministic table, decisions are not uniquely determined by the conditions. Instead, a subset of decisions is defined which could be taken under circumstances determined by conditions.

From the decision table a set of decision rules can be derived. Let $U|IND^s(C)$ and $U|IND^v(C)$ be families of all C^s - and C^v - elementary sets called strict and weak condition classes in DT^C , denoted by Y_i^s ($i=1, \dots, k_s$) and Y_i^v ($i=1, \dots, k_v$), respectively. Let, moreover, $U|IND^s(D)$ and $U|IND^v(D)$ be families of all D^s - and D^v -elementary sets called strict and weak decision classes in DT^C , denoted by X_j^s ($j=1, \dots, n_s$) and X_j^v ($j=1, \dots, n_v$), respectively.

$Des_C(Y_i^s) \Rightarrow Des_D(X_j^s)$ is called strict (C,D)-decision rule, while $Des_C(Y_i^v) \Rightarrow Des_D(X_j^v)$, weak (C,D)-decision rule in DT^C . The rules are logical statements (if... then...) relating descriptions of (strict and weak) condition and decision classes.

The set of (strict and weak) decision rules for each (strict and weak) decision class X_j^s ($j=1, \dots, n_s$) and X_j^v ($j=1, \dots, n_v$) is denoted by $\{r_{ij}^s\}$ and $\{r_{ij}^v\}$, respectively:

$$\{r_{ij}^s\} = \{Des_C(Y_i^s) \Rightarrow Des_D(X_j^s) : Y_i^s \cap X_j^s \neq \emptyset, i=1, \dots, k_s\}$$

$$\{r_{ij}^v\} = \{Des_C(Y_i^v) \Rightarrow Des_D(X_j^v) : Y_i^v \cap X_j^v \neq \emptyset, i=1, \dots, k_v\}$$

Rule r_{ij}^s (resp. r_{ij}^v) is deterministic iff $Y_i^s \subseteq X_j^s$ (resp. $Y_i^v \subseteq X_j^v$)

X_j^v), and r_{ij}^s (resp. r_{ij}^v) is non-deterministic otherwise. It means that decision rules are strict or weak, and, moreover, certain or possible.

The set of strict (resp. weak) decision rules for all strict (resp. weak) decision classes is called strict (resp. weak) decision algorithm.

7. ILLUSTRATIVE EXAMPLE

In order to illustrate the introduced generalizations, let us consider a small didactic example of a hypothetical information system. The system describes 12 clients who have got an approximately equal credit in a bank and now should pay back their debts in instalments. On the basis of dossiers, the clients are characterized by three condition attributes:

- c_1 - value of fixed capital,
- c_2 - value of sales in the year preceding the application,
- c_3 - kind of activity.

Attributes c_1 and c_2 are quantitative ones, while attribute c_3 is a qualitative one with three possible values.

Decision attribute d makes a dichotomic partition of the set of clients: $d=0$ if the client pays back the debt, and $d=1$ otherwise. The decision table DT with original values of attributes is shown in Table 1. It is clear that $C=\{c_1, c_2, c_3\}$ and $D=\{d\}$.

The values of quantitative attributes have been translated into qualitative terms (low, medium, high) using the norms presented in Fig. 2. The figure indicates the boundary values of subintervals, the thresholds on both sides of the boundary

values, and the codes of qualitative terms. The coded decision table DT^c is shown in Table 2.

Table 1. Decision table DT with original values of attributes

U \ Q	c_1	c_2	c_3	d
x_1	43	78	0	0
x_2	54	75	0	1
x_3	124	50	1	0
x_4	102	65	1	1
x_5	98	80	2	1
x_6	88	102	2	1
x_7	130	57	0	0
x_8	128	92	1	1
x_9	82	59	1	0
x_{10}	134	103	2	1
x_{11}	58	55	0	0
x_{12}	126	71	1	1

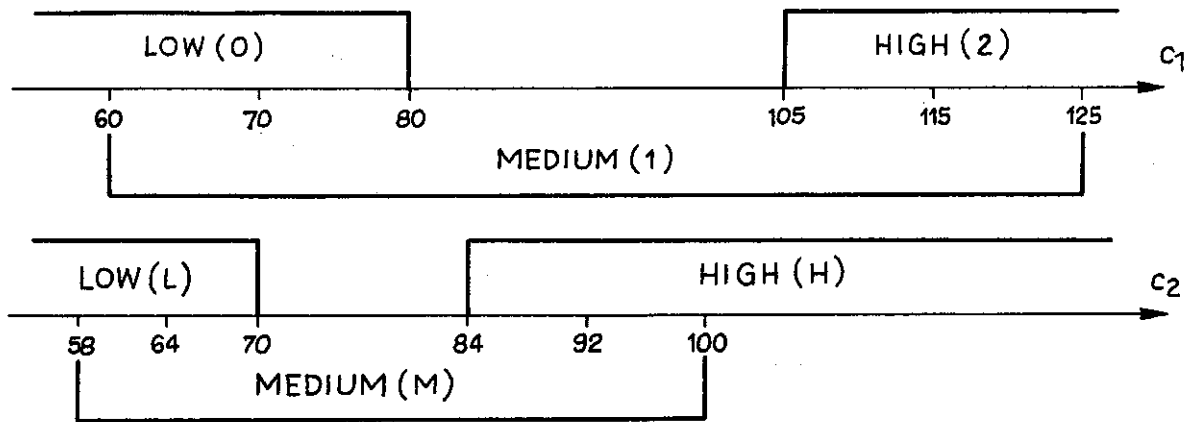


Fig. 2. Transformation of quantities into qualitative terms for attributes c_1 and c_2 (norms)

Table 2. Coded decision table DT^c

U \ Q	c_1	c_2	c_3	d
x_1	0	M	0	0
x_2	0	M	0	1
x_3	1, 2	L	1	0
x_4	1	L, M	1	0
x_5	1	M	1	1
x_6	1	H	2	1
x_7	2	L	2	0
x_8	2	M, H	1	1
x_9	1	L, M	1	0
x_{10}	2	H	2	1
x_{11}	0	L	0	0
x_{12}	2	M	1	1

The rough sets analysis of DT^c using strict and weak indiscernibility relations gives the following results.

- As strict and weak decision classes are identical, we will drop for them superscripts s and w:

$$X_0 = \{x_1, x_3, x_4, x_7, x_9, x_{11}\}, X_1 = \{x_2, x_5, x_6, x_8, x_{10}, x_{12}\}$$

$$\text{Classification } \mathcal{X} = \{X_0, X_1\}$$

- C^s -elementary sets:

$$\{x_1, x_2\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_{10}\}, \{x_{11}\}, \{x_{12}\}$$

- C^w -elementary sets:

$$\{x_1, x_2\}, \{x_3, x_4, x_9\}, \{x_4, x_5, x_9\}, \{x_6\}, \{x_7\}, \{x_8, x_{12}\}, \{x_{10}\}, \{x_{11}\}$$

- Strict approximations of class X_0 :

$$\underline{C}^s X_0 = \{x_7\} \cup \{x_{11}\}$$

$$\bar{C}^s X_0 = \{x_1, x_2\} \cup \{x_7\} \cup \{x_{11}\}$$

$$\text{Bn}_C^s(X_0) = \{x_1, x_2\}$$

- Weak approximations of class X_0 :

$$\underline{C}^w X_0 = \{x_3, x_4, x_9\} \cup \{x_7\} \cup \{x_{11}\}$$

$$\bar{C}^w X_0 = \{x_1, x_2\} \cup \{x_3, x_4, x_9\} \cup \{x_4, x_5, x_9\} \cup \{x_7\} \cup \{x_{11}\}$$

$$\text{Bn}_C^w(X_0) = \{x_1, x_2\} \cup \{x_4, x_5, x_9\}$$

- Strict approximations of class X_1 :

$$\underline{C}^s X_1 = \{x_5\} \cup \{x_6\} \cup \{x_{10}\} \cup \{x_{12}\}$$

$$\bar{C}^s X_1 = \{x_1, x_2\} \cup \{x_5\} \cup \{x_6\} \cup \{x_{10}\} \cup \{x_{12}\}$$

$$\text{Bn}_C^s(X_1) = \{x_1, x_2\}$$

- Weak approximations of class X_1 :

$$\underline{C}^w X_1 = \{x_6\} \cup \{x_8, x_{12}\} \cup \{x_{10}\}$$

$$\bar{C}^w X_1 = \{x_1, x_2\} \cup \{x_4, x_5, x_9\} \cup \{x_6\} \cup \{x_8, x_{12}\} \cup \{x_{10}\}$$

$$\text{Bn}_C^w(X_1) = \{x_1, x_2\} \cup \{x_4, x_5, x_9\}$$

- Strict and weak accuracy of approximation of \mathfrak{X} :

$$\pi_C^s = 0.6 \qquad \pi_C^v = 0.53$$

- Strict and weak quality of approximation of \mathfrak{X} :

$$\gamma_C^s = 0.5 \qquad \gamma_C^v = 0.75$$

- The indicator of an influence of overlapping:

$$\xi_C = 0.42$$

It means that 5 on 12 objects enter the C^t -boundary of \mathfrak{X} because of the overlapping.

- \mathfrak{X}^s - and \mathfrak{X}^v -reducts of C :

$$RED_{\mathfrak{X}}^s(C) = R_1^s \cup R_2^s, \text{ where } R_1^s = \{c_1, c_2\}, R_2^s = \{c_2, c_3\}$$

$$CORE_{\mathfrak{X}}^s(C) = R_1^s \cap R_2^s = \{c_2\}$$

$$RED_{\mathfrak{X}}^v(C) = R^v, \text{ where } R^v = \{c_1, c_2\}$$

$$CORE_{\mathfrak{X}}^v(C) = R^v = \{c_1, c_2\}$$

So, for strict and weak approximation of decision classes two condition attributes are sufficient. In the case of strict approximation, either c_1 or c_3 is superfluous, while c_2 cannot be removed without decreasing the quality of approximation. In the case of weak approximation, only c_3 is superfluous. Thus, in order to derive strict and weak decision algorithms, it is sufficient to consider the reduced decision table with c_1 and c_2 as the only condition attributes.

- $(c_1, c_2)^s$ -elementary sets (strict condition classes) are equal to C^s -elementary sets.

- $(c_1, c_2)^v$ -elementary sets (weak condition classes):

$$\{x_1, x_2\}, \{x_3, x_4, x_9\}, \{x_3, x_7\}, \{x_4, x_5, x_9\}, \{x_6\}, \{x_8, x_{10}\}, \\ \{x_8, x_{12}\}, \{x_{11}\}.$$

• strict decision algorithm:

- strict rule #1: if $c_2=L$ then $d=0$
- strict rule #2: if $c_1=0$ and $c_2=M$ then $d=0$ or 1
- strict rule #3: if $c_2=H$ then $d=1$
- strict rule #4: if $c_1=1$ and $c_2=M$ then $d=1$
- strict rule #5: if $c_1=2$ and $c_2=M$ then $d=1$

Strict rule #2 is non-deterministic (possible) while four others are deterministic (certain).

• weak decision algorithm:

- weak rule #1: if $c_1=\underline{1}/2$ and $c_2=\underline{L}/M$ then $d=0$
- weak rule #2: if $c_1=1/\underline{2}$ and $c_2=L$ then $d=0$
- weak rule #3: if $c_1=0$ and $c_2=L$ then $d=0$
- weak rule #4: if $c_1=0$ and $c_2=M$ then $d=0$ or 1
- weak rule #5: if $c_1=1$ and $c_2=L/\underline{M}$ then $d=0$ or 1
- weak rule #6: if $c_1=1$ and $c_2=H$ then $d=1$
- weak rule #7: if $c_1=2$ and $c_2=M/\underline{H}$ then $d=1$
- weak rule #8: if $c_1=2$ and $c_2=\underline{M}/H$ then $d=1$

Note: $c_1=\underline{1}/2$ means that c_1 falls into subinterval 1 or into the overlapping zone of subintervals 1 and 2; $c_2=M/\underline{H}$ means that c_2 falls into the overlapping zone of subintervals M and H or into subinterval H, etc.

Weak rules #4 and #5 are non-deterministic (possible) while six others are deterministic (certain).

The strict decision algorithm could be red as follows:

All clients showing the value of sales lower than 58 certainly don't pay back the debt, while those showing the value of sales higher than 100 certainly pay, independently on the value of their fixed capital. Clients with the value of sales in between 70 and 84 pay back the debt only if the value of their fixed capital is higher than 80, otherwise it is not certain if they pay or not. The kind of activity doesn't influence the clients' ability of paying back the debt.

Interpretation of the weak decision algorithm could be the following:

All clients showing the value of sales lower than 58 certainly don't pay back the debt, independently on the value of their fixed capital. Clients with the value of sales in between 58 and 70 and the value of their fixed capital in between 105 and 125 also certainly don't pay, while it is not certain if those with the value of sales in between 70 and 80 pay back or not. It is certain, however, that all clients with the value of sales higher than 84 pay back the debt. Again, the kind of activity doesn't influence the clients' ability of paying back the debt.

It can be seen that consideration of overlapping subintervals made possible a more flexible interpretation of the decision algorithm.

8. CONCLUSIONS

Overlapping subintervals defined for transformation of quantitative data into qualitative terms require a generalization of the indiscernibility relation used for definition of a rough set. Distinction between strict and weak indiscernibility relation requires in turn generalization of other concepts of the rough sets theory. The paper provides such a generalization and ends with an illustrative example of its application.

Let us remark that the next step of the generalization could consist in considering the overlapping subintervals as fuzzy subintervals.

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REFERENCES

- [1] D.Dubois, H.Prade. Putting rough sets and fuzzy sets together. In: R.Slowinski (ed.), *Intelligent Decision Support - Handbook of Applications and Advances of the Rough Sets Theory*. Kluwer Academic Publishers, Dordrecht, 1992.
- [2] J.Fibak, Z.Pawlak, K.Slowinski, R.Slowinski. Rough sets based decision algorithm for treatment of duodenal ulcer by HSV. *Bull.Pol.Acad. Sci., biol.ser.* 34 (1986) 227-246.
- [3] E.Krusinska, R.Slowinski, J.Stefanowski. Discriminant versus rough sets approach to vague data analysis. *J.Applied Stoch. Models and Data Anal.* 8 (1992) no.2.

- 21
- [4] J.Krysinski. Rough sets approach to analysis of structure-activity relationship of quaternary imidazolium compounds. *Arzneimittel-Forschung / Drug Research* 40 (1990) 795-799.
 - [5] R.Nowicki, R.Slowinski, J.Stefanowski, Rough sets analysis of diagnostic capacity of vibroacoustic symptoms. *J. Computers & Mathematics with Applications* (to appear).
 - [6] Z.Pawlak. Rough sets. *Int.J.Information and Computer Sci.* 11 (1982) 341-356.
 - [7] Z.Pawlak. *Rough Sets - Theoretical Aspects of Reasoning about Data*. Kluwer Academic Publishers, Dordrecht, 1991.
 - [8] Z.Pawlak. Rough sets and fuzzy sets. *Fuzzy Sets and Systems* 17 (1985) 99-102.
 - [9] Z.Pawlak, K.Slowinski, R.Slowinski. Rough classification of patients after highly selective vagotomy for duodenal ulcer. *Int.J.Man-Machine Studies* 24 (1986) 413-433.
 - [10] R.Slowinski. Rough sets with strict and weak indiscernibility relations. *Proc. IEEE Int. Conference on Fuzzy Systems*, San Diego, California, 1992, 695-702.
 - [11] R.Slowinski (ed.). *Intelligent Decision Support - Handbook of Applications and Advances of the Rough Sets Theory*. Kluwer Academic Publishers, Dordrecht, 1992.
 - [12] K.Slowinski, R.Slowinski. Sensitivity analysis of rough classification. *Int.J.Man-Machine Studies* 32 (1990) 693-705.
 - [13] K.Slowinski, R.Slowinski, J.Stefanowski. Rough sets approach to analysis of data from peritoneal lavage in acute pancreatitis. *Medical Informatics* 13 (1989) 143-159.
 - [14] R.Slowinski, J.Stefanowski. Rough classification in incomplete information systems. *Mathl. Comput. Modelling* 12 (1989) 1347-1357.
 - [15] A.Skowron, J.Grzymala-Busse. *From the Rough Set Theory to the Evidence Theory*. ICS Research Report 8/91, Institute of Computer Science, Warsaw University of Technology, Warsaw, 1991.